# Using Dynamic Software to Teach Mathematical Concepts: The Cases of GeoGebra and Microsoft Mathematics

*Seyum Tekeher Getenet*
e-mail: Seyum.Getenet@usq.edu.au
University of Southern Queensland
School of Teacher Education and Early Childhood, Queensland, Australia

***Abstract***

*The findings presented here are part of a larger study, which documented the design, development and refinement of a professional development program to improve teacher educators' effective use of technology to teach mathematics. This paper describes the use of free software to teach the area of a unit circle and then estimate the decimal value of pi (π), and graphs of logarithmic functions. Data comprised video-recorded lessons audited using an observation checklist, and interview data provided by two teacher educators and five pre-service teachers who were in the class of learners. Data were analysed using a quantitative and thematic grouping of the qualitative data. In this study, although students often find mathematics concepts abstract and difficult to understand, it was found that using technologies made the lessons more engaging by enabling trial, improvement and experimentation with the tasks. Furthermore, this approach fostered learners' peer exchange by providing support for exploration and consequent sharing of discoveries. For such effective pedagogy, the study also found that teacher educators needed an understanding of their learners' prior knowledge of the technology, and equally the learners should have prior knowledge to be able to use the specific software effectively. The study further provided an example of how free software can be used to explore basic mathematics concepts.*

## 1. Introduction

University, college, and school teachers are motivated to integrate Information Communication Technology (ICT) into their mathematics teaching to achieve better learning outcomes on the assumption that the use of ICT in teaching can lead to significant positive pedagogical outcomes [7]. Studies also have shown that ICT can support constructivist pedagogies, whereby students use technology to explore and to reach an understanding of concepts [7]. Integration of ICT in teaching thus has become a key component of educational reform agendas across the world. For example, countries in Africa strongly endorse ICT use as an essential component of innovative student-centred pedagogy to improve the quality of education [17]. In Ethiopia, the site of this study, the school curriculum and education system have been characterised as low in quality [10]. In particular, there is evidence that the quality of students' performance in mathematics is stubbornly inadequate at all levels [1]. This was evident in the national learning assessment which is conducted every four years indicating that students' performance in mathematics is lower compared to other subjects (see [14]). There is, therefore, considerable support throughout the system, including in the higher education sector, to improve the quality of education. As part of this endeavour, efforts have been made to encourage teachers at all levels to integrate ICT in their teaching particularly in science and mathematics in Ethiopia. These efforts include national initiatives that encourage teacher educators to use ICT in their teaching as a means to improve the quality and to promote equity in education [28]. This study aligns with that effort in exploring the evident knowledge and practices of Ethiopian mathematics teacher educators when they used free software, namely GeoGebra and Microsoft Mathematics (MSM) to teach the area of a unit circle to estimate the value of π, and graphs of logarithmic functions.

## 2. Teaching Mathematics with ICT

There is a great deal of mathematics specific software available [19] that can provide opportunities for active learning and enhanced student engagement [7]. Although the integration of ICT in teaching has many generic aspects, as described for example by [24], there is a need to consider the use of technology in particular subjects such as mathematics and, indeed, about specific mathematics content [21]. In addition, the affordances of specific technological tools which was considered in this study as one of the criteria to select technologies, influences a section of possible teaching approaches and an appropriate pedagogies [30]. For instance, simulations and animations enable students to vary a selection of input parameters, observe how each affects the system under study, and interpret the output results through an active engagement of hypothesis-making, and ideas testing. They can explore combinations of factors and observe their effects on the evolution of the system under study. For such purposes, there are specific software including GeoGebra, MSM, Maxima, STELLA, and spreadsheets to facilitate mathematics learning. For example, GeoGebra has been rapidly gaining popularity among teachers and researchers around the world, because it is easy-to-use and combines many aspects of different mathematical packages. In addition, because of its open-source nature, which allows it to be downloaded for multiple platforms or to be launched directly from the Internet, an extensive user community has developed around it [20]. This user community could be helpful to share ideas about its effectiveness to teach mathematics. As a result, various studies encouraged teachers to use GeoGebra to teach mathematical concepts taking into account some of its challenges (see [12] and [20]). Although some challenges in using GeoGebra have been identified ([12] and [20]), such as, the need to have previous experience to enter algebraic commands in the input box and limited research on the impact of GeoGebra on teaching and learning of mathematics, a recent study showed that GeoGebra had advantages in enabling students to discover mathematical concepts independently [26]. Most importantly, it can help to foster experimental, problem-oriented and discovery learning of mathematics (see [20] and [26]). Such software application could also be used as alternative solutions in learning mathematical concepts such as geometry not only to attract and motivate students, but more to provide opportunities for students to develop skills in understanding, reasoning, and problem solving skills [12].

Similarly, MSM is free software that can be used to help students to achieve an understanding of a range of mathematical concepts by helping students to visualise the effects of changed parameters as it was used in this study. Different from GeoGebra, the MSM interface allows for solving problems with minimal syntax instruction [33] and facilitates animation. The use of the 'Animate' command found within MSM can also aid discovery-style lessons [31]. It offers, for example, visualisation of shapes of graphs of families of logarithmic functions by learners input of bases, b, between $b > 1$, and $0 < b < 1$. However, [22] found that students struggled greatly with the concept of logarithmic functions and sketching their corresponding graphs, and [2] showed that students learn the characteristic properties of families of functions more effectively using technology than without using any ICT. Both GeoGebra and MSM can be used in teaching and learning mathematics from middle school through to college level.

In Ethiopia, financial constraints often mean that free software is preferable. Accordingly, the potential for using GeoGebra and MSM to teach and learn mathematics in Ethiopia is considerable.

## 3. Teachers' Knowledge for Technology Integrated Teaching

Much research has been undertaken aimed at understanding the knowledge teachers need for successful integration of technology in teaching (see [7] and [29]). It has been argued that teachers' knowledge of ICT is not the only requirement to effectively use ICT in teaching; sound pedagogical and content knowledge are also critical to success [7]. [37]'s notion of Pedagogical Content Knowledge (PCK) is thus relevant but requires the additional dimension of technological knowledge. The incorporation of a technology component into PCK resulted in the development of the notion of "Technological Pedagogical Content Knowledge" (TPACK) [30]. According to the TPACK framework, Technological Knowledge (TK), Pedagogical Knowledge (PK) and Content Knowledge (CK) can reinforce each other to realise the advantages afforded by technology in the teaching and learning process. The TPACK framework is presented in Figure 1 followed by a definition of each knowledge construct in Table 1.

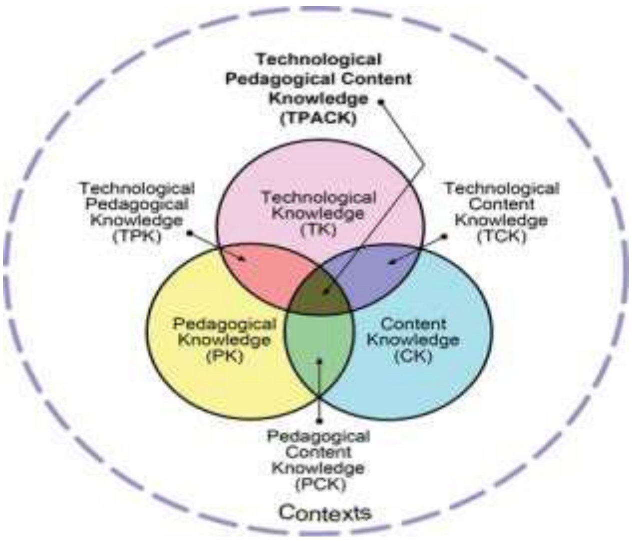


*Figure 1.* Technological Pedagogical Content Knowledge framework. Reprinted by permission of the publisher from *tpack.org*, by [30], Retrieved from http://tpack.org/. Copyright © 2012 by tpack.org.

The combination of technology, pedagogy and content result in four additional composite knowledge types, namely: Technological Content Knowledge (TCK), Technological Pedagogical Knowledge (TPK), Pedagogical Content Knowledge (PCK) and TPACK at the intersection of all three. According to [25] and [32], the ability of teachers to establish the relationship between content, pedagogy and technology depends largely on the way that they have been taught to integrate technology into teaching and the way these components are treated in Proffesional Development (PD) programs. On which the larger study emphasised developing teacher educators' TPACK through a PD program.

Table 1
*Domains of Technological Pedagogical Content Knowledge [30]*

| Component | Description |
|---|---|
| TK | Knowledge needed to use a particular technology. This could be, for example, using a particular software program and installing or removing it. |
| PK | Knowledge about process and practices of teaching. This knowledge includes, for example, students' learning styles, classroom management, students' evaluations and lesson planning. |
| CK | Knowledge of a subject matter to be taught. This knowledge demands understanding core principles, facts, theories, procedures and concepts of a particular subject matter. |
| PCK | Knowledge of how particular pedagogical approaches are suited to teaching particular content and vice versa. |
| TCK | Knowledge of teachers in which technology and content interact in effective teaching. It includes teachers' understanding of how subject matter can be changed by the use of technology. |

| | |
|---|---|
| TPK | Knowledge of how to use various technologies with different pedagogical approaches. It involves recognising and making use of the affordances of technologies and choosing pedagogical approaches that fit particular technologies and vice versa. |
| TPACK | Knowledge that is more than the sum of its three components (content, pedagogy, and technology). It is the basis of effective teaching with the application of technology and requires an understanding of pedagogical techniques that use technologies in constructive ways to assist students to overcome difficulties and to learn content effectively. |

In addition to the above 7 TPACK constructs, contexts in Figure 1 included teachers' knowledge of students' existing knowledge of the content and relevant technologies [30]. The TPACK framework has been used to describe what teachers need to know to integrate technology into their teaching effectively. As used in this study, the TPACK framework is also useful to think about how teachers might develop knowledge for technology integrated teaching and evaluate teachers' effectiveness in their technology integrated teaching [29]. In line with [29]'s recommendation, the TPACK framework was used in this study to evaluate teacher educator's effectiveness to use technology to teach graphs of the logarithmic functions and are of a circle to estimate the decimal value of π.

## 4. Context of the Study

There were 32 Colleges of Teacher Education (CTEs) in Ethiopia responsible for awarding diplomas to second cycle primary school teachers administered by regional States. The research sites were the departments of mathematics in two Ethiopian CTEs. Both CTEs were selected as study sites because the researcher had experience working with staff members of these CTEs and the CTEs had similar organisational structures. The CTEs prepared first (Grades 1 to 4 or aged from 7 to 10) and second (Grades 5 to 8 or aged from 11 to 14) cycle primary school teachers. The second cycle primary school teachers, on whom this study is focused, specialised in a particular subject including mathematics, physics, biology or chemistry. Second cycle primary mathematics pre-service teachers (PSTs) were required to attend the basic mathematics and professional courses in a three-year program. Mathematics courses include Fundamental Concepts of Algebra, Plane Geometry, Basic Mathematics I and II, and Introduction to Calculus. The professional courses included Methods of Teaching Mathematics. The PSTs in the study were enrolled in a Plane Geometry unit and Basic Mathematics II. The report involved two case studies (Case 1 and Case 2). Case 1 involved a video-recorded lesson totalling 2 hours which documented a teacher educator's teaching using MSM. In this case, PSTs were those who were finishing their first year in the program. They were enrolled in Basic Mathematics II, the content of which included graphs of logarithmic functions. Most of them were aged between 18 and 24 years. Twenty nine PSTs (18 males and 11 females) participated in the observation part of Case1. In Case 2, PSTs finishing their second year of a 3-year program were involved. They were enrolled in a Plane Geometry unit, the content of which included estimating the decimal value of π using the area of a unit circle. Thirty one PSTs participated in the observation part of Case 2. Interview data was aggregated from those PSTs who referred to this lesson during the interview for the larger study.

A short version of Case 1 was presented in a conference [15] which illustrated the use of specific software to teach a mathematics concept for understanding. The current paper includes Case 2 and additional dimensions which looked at the knowledge required to teach technology integrated mathematics lessons and the teacher educators and PSTs perception towards the lessons.

## 5. Method of the Study

This section reports the procedure of the study, participants, data collection and data analysis processes.

### 5.1. Procedure

The study involved two non – consecutive lessons on different topics. The video-recorded lessons, totalling 2 and 1.5 hours, and involving the use of MSM and GeoGebra, were taught by two teacher educators. The purpose of using MSM and GeoGebra was to teach the graphs of logarithmic functions and area of a unit circle to estimate the decimal value of π respectively. Once the PSTs were familiar with the menus and toolbars of the software, they learned how to graph logarithmic functions and used the area of a unit circle to estimate the decimal value of π.

While using MSM, PSTs were asked to work in groups of three or four to illustrate properties of graphs of logarithmic functions and the questions shown in Figure 2 were provided to guide their work while Teacher Educator 1 was using MSM.

1) Sketch the graphs of the following logarithmic functions:
   a. $f(x) = \log_2 x$
   b. $f(x) = \log_5 x$
   c. $f(x) = \log_{\frac{7}{2}} x$
   d. $f(x) = \log_{\frac{2}{3}} x$
   e. $f(x) = \log_{\frac{1}{2}} x$
2) Describe the shapes of the graphs when $b > 1$, and $0 < b < 1$
3) Describe properties of the graphs listed in question 1
   a. Common properties
   b. Describe the graphs when $x > 1$, and $0 < x < 1$, $x = 1$

*Figure 2*. Questions explored using MSM

In Case 2, firstly, the PSTs were asked to find the lengths of the perimeter and apothem (Apothem is a line from the centre of a regular polygon that meets any of its sides at a right angle) of regular polygons using GeoGebra, working in groups of three or four. The questions shown in Figure 3 were provided to guide their work.

Compare the areas of regular polygon inscribed in a circle of radius 1 cm and the circle itself. The formula for areas of a regular polygon is

$$A(Poloygon) = \frac{n \times side \times apothem}{2} = \frac{Perimeter \times Apothem}{2} \text{ and } A(c) = \pi r^2, \text{ where}$$

apothem of a regular polygon is defined as a line segment from the centre to the midpoint of one of its sides.

1. What do you observe as $n$ increases from 3 to 30 in the shape of the polygon?
2. What will be the relationship between the area of the regular polygon and the unit circle as $n$ increase?
3. Compare the radius of the circle and the apothem and approximate value of π.

*Figure 3.* Questions explored using GeoGebra.

Secondly, they were instructed by the teacher educator to use the formula $A(Poloygon) = \frac{n \times side \times apothem}{2} = \frac{perimeter \times apothem}{2}$ to calculate the area of a regular polygon with *n* sides. Finally, they were asked to record the result in a table provided (see Table 2) and answer the questions shown in Figure 3.

At the beginning of the lesson, the teacher educator showed how to find the area, perimeter and apothem of the first three polygons using GeoGebra as shown in Table 2.

Table 2

*Area of selected regular polygons inscribed in a circle using GeoGebra*

| Polygon | Number of sides | Polygon area ($cm^2$) | Perimeter (cm) | Apothem (cm) |
|---|---|---|---|---|
| Triangle | 3 | 1.29903 | 5.20 | 0.71 |
| Square | 4 | 2.00000 | 5.74 | |
| Pentagon | 5 | 2.37764 | 5.90 | |
| | n | ?? | ?? | |

Then the teacher educator used GeoGebra to draw an inscribed square in a unit circle and display their areas as indicated in Figure 4 to compare its area with the area of the unit circle.

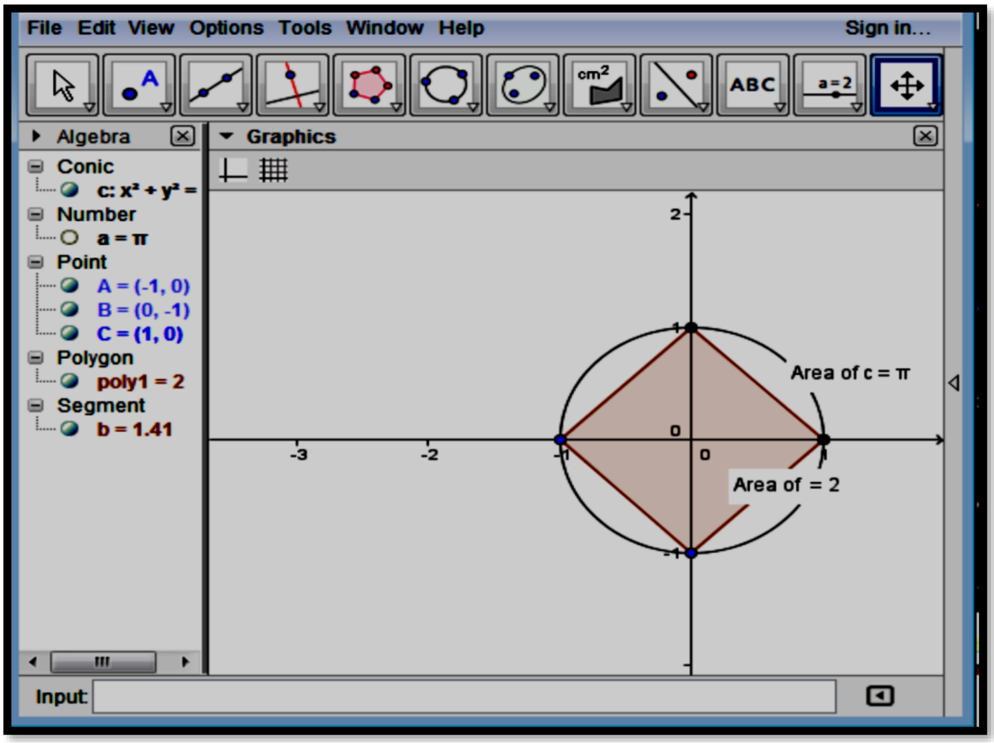


*Figure 4.* Area of a unit circle and inscribed regular polygon using GeoGebra.

Following the lessons of the two teacher educators, semi-structured, audio-recorded, individual interviews were conducted with five PSTs and the two teacher educators. The two lessons were taught to two different groups of PSTs (PSTs finishing their first year and PSTs finishing their second year respectively) by Teacher Educator 1 and Teacher Educator 2.

### 5.2. Participants

Participants were PSTs finishing their first and second year of a three-year program in Ethiopian CTE. Two male mathematics teacher educators, who were willing for their lessons to be video recorded, were observed while teaching the lessons. They had participated in a professional learning program over the previous 5 months by the researcher, aimed at encouraging them to use technology to teach mathematics. The professional learning program includes sessions focused on the pedagogical use of technology, using TPACK framework [30] to design lessons and using easily accessible web based technology. Both had 15 years of teaching experience in CTE with limited prior experience in using ICT in mathematics teaching, and their qualifications were a Bachelor degree in mathematics education and a Master degree in algebra. Teacher Educator 1 taught the properties of logarithmic functions using MSM for 2 hours whereas Teacher Educator 2 used GeoGebra for 1.5 hours to teach the area of a circle and estimating the decimal value of π.

Of the participant PSTs of both cases, four male PSTs and one female PST provided the data for this particular study. The PSTs were purposefully selected with the help of the teacher educators as being able to express their ideas clearly in English. The five PSTs were also enthusiastic and keen to use GeoGebra and MSM in their future teaching of mathematics, and for learning mathematical concepts in their current units but had never used the software before these lessons.

### 5.3. Data Collection

The data collection instruments were interviews and observations of teaching practices in the classroom. Both qualitative and quantitative data were collected. The following sections detail the research instruments.

5.3.1. **Observation checklist**. The two lessons were video recorded and evaluated by the researcher using a pre-designed checklist (see the appendix). The checklist was adapted from [3] and designed to evaluate mathematics teacher educators' technology integrated mathematics teaching practices based on the seven TPACK constructs (see description for each construct in Table 1). The checklist comprised 20 items measuring the TPACK constructs using a scale (3 = observed, 2 = partly observed, and 1= not observed). It also included space to describe observed practices. For example, if a teacher educator scored a modal value of 2 from the 3 items addressing the TK construct, the teacher educator had no technical problems in using the technology but was observed having some irregularities using the technology to teach the mathematical concept. A modal value of 3 (observed) meant they used the technology effectively without any challenges or irregularities. Two mathematics educators with experience in the analysis of observational data also reviewed the videos and verified the coding to the extent possible given that the lessons were not conducted purely in English. This process was used to enhance the reliability and validity of the study [5].

5.3.2. **Interviews**. The interviews took about 25 minutes for each interviewee and covered a range of topics. As the study adopted a semi-structured interview approach, it supported to prepare the relevant questions before the interviews. As recommended by [11], these questions were few but allowed for in-depth discussion of the issues. The relevant questions for this study to PSTs were: What did you like about the lessons? Did the technology help your learning? In what ways? The interview questions to the teacher educators asked for their views on the lessons they taught with MSM and GeoGebra and their previous teaching of, for example, graphs of logarithmic functions (such as how the lesson engaged learners? What did you dislike and like about these lessons? Why?). The questions were designed to allow in-depth probing of PSTs' and teacher educators' views associated with the lessons delivered using GeoGebra and MSM. Also, the interviews were designed in English but participants had the opportunity to express their ideas with a mix of English and the local language (Amharic). For the purpose of this research, however, the English translation is reported.

### 5.4. Data Analysis

The interview data from the PSTs and teacher educators and the notes from the video recordings were analysed to identify themes as described by [9]. For this study, parts of the interviews relevant to using GeoGebra and MSM were included. The observation data were rated by watching video recorded lessons and calculating the modal value for each TPACK construct on the use of GeoGebra and MSM by the teacher educators. Notes were made in the spaces provided in the observation checklist when evidence was observed. Consistent with the advice of [5], the analysis emphasised aspects of ICT use known to be relevant, such as how the PSTs interacted with the GeoGebra and MSM, specifically their use of the GeoGebra and MSM, and how they worked to make sense of their learning. These were analysed by thematically categorising into clusters that addressed the same issue. First, each set of data were transcribed and imported into Excel for filtering purpose and again transported to Microsoft Word to highlight the identified themes. The transcripts were read several times to obtain a sense of the whole. As suggested by [9] the qualitative data analysis used an inductive approach. Instead of stating hypotheses at the beginning, the researcher generated categories from the participants' responses as themes emerged. The themes were identified by searching through texts and marking them with different coloured pens. A consistent code name was given to each of the teacher educator (such as Teacher Educator 1) and pre-service teacher (such as PST1) in the report. The mode was used to analysis the teacher educators' TPACK as observed in the classroom. Because the scale used had only three points, the mode was used to better analysis their practices from the observed data [18].

## 6. Results

The results are presented in two cases (Case 1 and Case 2). Case 1 refers to Teacher Educator 1 teaching graphs of a logarithmic function using MSM and Case 2 refers to Teacher Educator 2 teaching the area of a unit circle to estimate the value of π using GeoGebra.

### 6.1. Case 1: Using MSM to Illustrate Graphs of Logarithmic Functions

Teacher Educator 1 described two methods he had previously used to teach sketching graphs of logarithmic functions. The first involved taking a simple logarithmic statement, switching it around to the corresponding exponential statement, and then figuring out the *x*-value needed for that exponent (*y*-value). The second, the T-chart method, is carried out by taking powers of the base of the function as *x*-values and finding the corresponding *y*-values. The teacher educator identified this method as preferred because it requires learners to know the procedures for finding the values of logarithmic functions. For example, to draw the graph of $\log_2(x)$, PSTs first list some values of *x* and *y* on the T-chart and then sketch the graph by connecting points as indicated in Figure 5. He acknowledged that this method was challenging for comparing multiple graphs on the same axes. For example, it is difficult to exactly identify which graph approaches the y axis when $x > 1$, and $0 < x < 1$. The results are presented in three area corresponding to the questions in Figure 3.

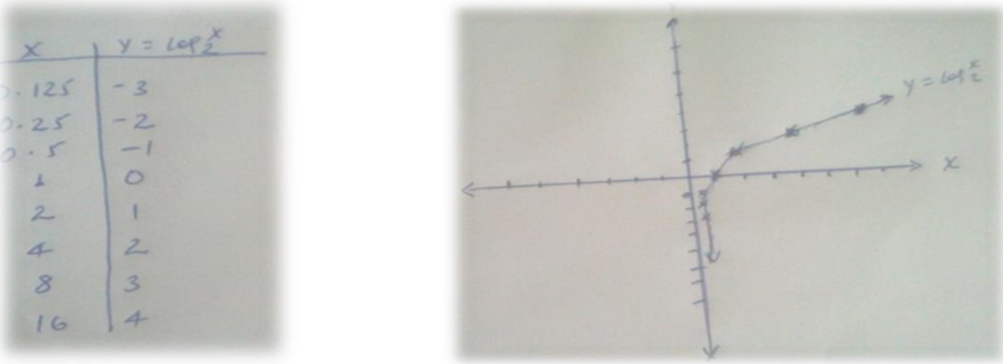

*Figure 5.* T-chart and graph for $f(x) = \log_2(x)$

Firstly, the teacher educator began by presenting the definition of the logarithmic function, y=log $_b(x)$, where $b$ is any number such that $b > 0$, $b \neq 1$ and $x > 0$. Using MSM, each group of PSTs was able to draw multiple graphs of logarithmic functions easily, with different colours, and on the same axes, as illustrated in Figure 6.

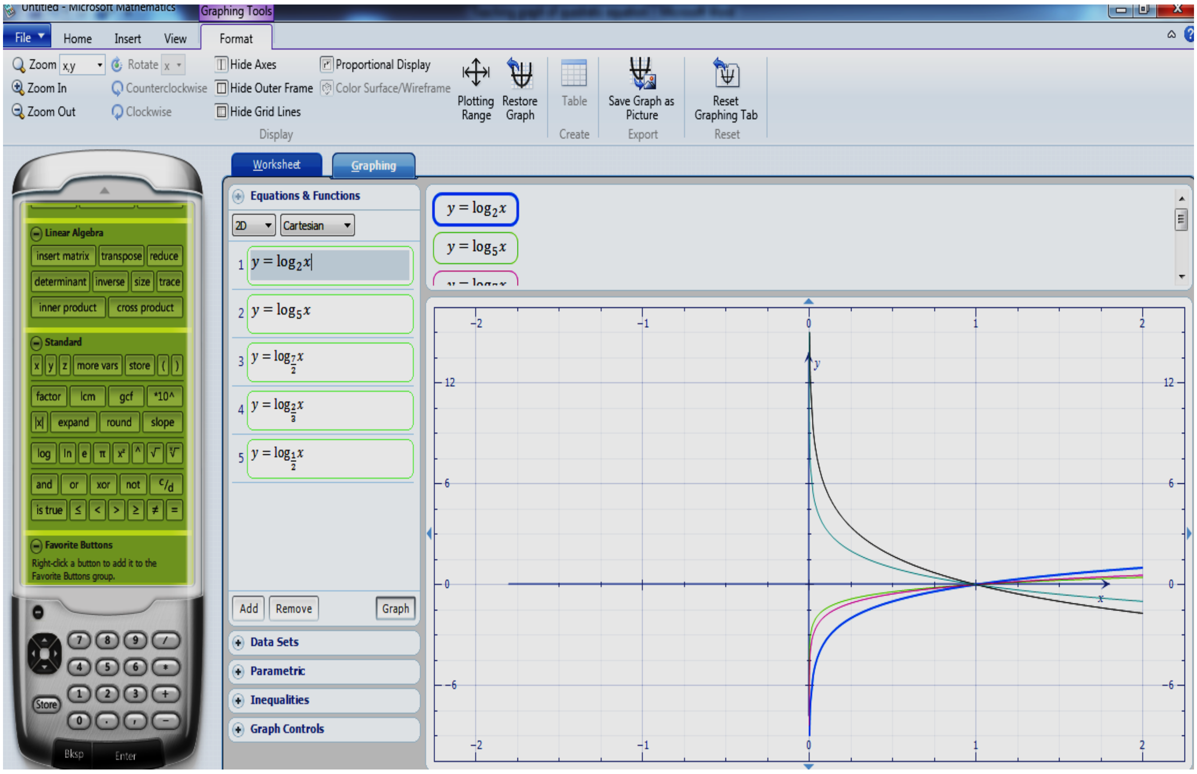


*Figure 6*. Graphs of some logarithmic functions created in MSM

They were required to write the equation in the "writing box" and click on the icon 'graph' to find the graph of the corresponding equation, and appeared to enjoy sketching the graphs. During the interview, PST 1 pointed to the effect of using the software on her engagement while admitting incomplete understanding of what was happening. She said:

> *The software helped me to easily sketch each graph on the same x-y axis with different colours; however, I don't know how it happens.* (PST 1)

This result was an indicative for the teacher educator to closely support all the PSTs groups to understand the concept through the process rather than letting them anticipate the output from the software application. For example, in the recorded video, one group of PSTs was anticipating the position of the graphs. The anticipation was based on the assumption that the graph of the basic log function such as y = log2(x) crawled up the positive side of the y-axis to reach the x-axis, with the line never going to the left of the limitation that $x$ must be greater than zero.

Secondly, the teacher educator described shapes of the graphs of logarithmic functions when $b > 1$, and $0 < b < 1$. The PSTs were able to describe the shapes of the graphs with a general equation $f(x) = \log_b(x)$ without sketching multiple graphs but rather by changing the value of $b$ between $b > 1$, and $0 < b < 1$ using the "Animate" feature of MSM to generate a movie of different graphs as $b$ changed. Alternatively, $b$ could be directly controlled by inputting a value. Using the animate icon, PSTs observed and described the shapes of logarithmic functions for values of $b$ between 0 and 2. In the video-recorded lesson, they appeared to recognise and appreciate the shape change when $b$ passes 1. During the interview, PST 3 indicated his interest in these animations features of MSM.

> *I liked the role of "animate" to clearly see the shape of the graphs of multiple logarithmic functions as the base b varies without sketching samples of multiple graphs.* (PST 3)

This form of observation was difficult to get from the traditional approach as the process doesn't clearly indicate the motional movement of the graphs as shown in Figure 5.
The PSTs appeared to recognise and appreciate the shape change when b is greater than1 because of using MSM.

PST1 described what she saw when $b$ passes 1 as:

> *By using the animate function I was able to understand the graph approached positive y- axis as $b < 1$, whereas, it approached negative y-axis as $b > 1$.* (PST1)

In the video recording, the PSTs have been arguing that the graphs never enter the second or third quadrants. This is because when 'x' is negative (second or third quadrant) there exist no real exponent combined with a positive base yielding a negative answer. This result indicated that the PSTs were prospectively relating logarithmic functions with exponential functions.

Thirdly, the teacher educator dealt with the concepts of describing the common properties of the graphs of logarithmic functions. The groups of PSTs readily identified that all logarithmic functions have the same general shape, with their graphs varying depending on the base and coefficients in the equation. During the lessons, PSTs were pointing to the graphs made using MSM to identify and describe the common properties of the logarithmic function in each category, $b > 1$, and $0 < b < 1$, for example, the fact that all have a vertical asymptote at $x = 0$, and cross the $x$-axis at $x = 1$. When interviewed, PST 2 described the usefulness of MSM as follows:

> *I liked the software which helped to graph all logarithmic functions on the same x-y axis with different colours. This helped me to list and understands the common properties of logarithmic function as the base, b varies*. (PST 2)

This suggestion clearly showed the advantage of the software being a multi task and showing all graphs at the same time which was also helpful to PSTs to analysis and compare the properties of each graph.

In addition, the PSTs used MSM to describe shapes of the graphs when $x > 1$, $0 < x < 1$, $x =1$. In this time, most groups of PSTs described the shapes of the graphs by observing the sketched graphs. However, one group was observed trying to identify the properties of the graphs through the 'Trace' function of MSM. The trace function varies the values of $x$ continuously and shows the value of $y$ for a given base $b$, as $x$ moves through a specified range of values. In this case PSTs identified the values of $y$ as $x$ moved between $x > 1$, $0 < x < 1$.

In a summary of this case, the teacher educator and the PSTs had a range of reflections on the use MSM. The PSTs expressed a range of perspectives on the use of MSM in learning about graphs of logarithmic functions. One of the PSTs had mixed feelings about using MSM, expressing a preference to use MSM and without using MSM. Although she recognised the significance of technology, she tended to believe that graphs of the logarithmic function should be first taught without using any ICT then later using MSM. This was the same PST who had admitted being unsure of how MSM produced the graphs. Another explained the advantage of MSM by comparing the lessons with his previous lessons. He said:

> *At first glance, the graph of the logarithmic function can easily be mistaken for that of the square root function when sketching manually. Both the square root and logarithmic functions have a domain limited to x values greater than 0. However, the logarithmic function has a vertical*

> *asymptote descending towards negative ∞ as x approaches 0, whereas the square root reaches a minimum y value of 0. This difference was clearly demonstrated by using MSM*. (PST 3)

Another PST indicated that MSM helped to externalise his reasoning, work at his own pace, and manage the complexity of the task scaffolding pen-and-paper skills. He said:

> *MSM complements my learning of graphs of logarithmic function by helping to visualise, understand, and animate to identify their properties. ... I liked the process as I was engaged and discussed with peers throughout the process and it was a different approach*. (PST 2)

It was apparent from this comment that that simply providing the graphs was not adequate to understand properties of the graphs rather facilitating PSTs' visualisation had an increased importance because PSTs can explore, solve, and communicate the mathematical concepts through the process.

The teacher educator described the role of MSM as follows:

> *The software was vital and complements PSTs' ability to discuss the problem by engaging PSTs in a small group guided by me. The discussion within their small groups was thought-provoking as they were engaged through manipulating the computer. I liked MSM as it complements my efforts by helping PSTs to visualise graphs of logarithmic functions as well as provoking active engagement by PSTs.* (Teacher Educator 1)

The software application provoked open discussion among PSTs while different shapes of graphs were formed by them. Understanding the reasons behind different shapes and properties were the point of discussion.

### 6.2. Case 2: Using GeoGebra to Teach Area of a Circle and approximate the decimal value of π

In this case, the PSTs used the table shown in Table 2 to compare the area of the regular polygon with the area of the unit circle as the number of sides of the polygon (*n)* increased. PSTs drew unit circles and then the inscribed polygon, varying the number of sides (*n*). They used tools in GeoGebra to find the perimeter and apothem (a line segment from the centre to the midpoint of one of its sides of a regular polygon) of the regular polygon as the number of sides, *n*, increased. During the lesson, there was an active discussion in each group, especially when comparing the areas of the circle and the inscribed polygon. During the interview, one of the five PSTs commented about this lesson as follows:

> *The lesson on finding in the area of a circle with the help of inscribed regular polygon using GeoGebra was engaging and supported [us] to estimate the value of π. We know how to find the area of a regular polygon when the perimeter and the apothem are given*. (PST 4)

Observations showed that some PSTs found using the software to sketch an inscribed regular polygon with *n* sides in a unit circle challenging. Groups of PSTs were comparing the area of a circle with that of the inscribed regular polygon regardless of the length of the radius of the circle. This occurred due to their limited prior knowledge of GeoGebra. They were unable to create a 'slider' which is useful to avoid typing over and over again for varying values of the number of sides of polygons. During the interview, the importance of prior knowledge was cited by PST 5. He said:

> *Rather than directly using a particular technology for teaching, it could be wise first to practice and [be] familiar with the technology. It takes most of our time, for example, to sketch inscribed a regular polygon in a unit circle using GeoGebra*. (PST 5)

Others have also been argued that since it is possible to find the area of any polygon by dividing it into triangles, it is also possible to approximate the area of a circle by increasing the number of sides of the inscribed polygon. This group of the PSTs concluded that the area of the inscribed polygon approached π as *n* increased. For example, for *n* = 3, the area is 1.2990 and for *n* = 200, the area is 3.1411. Inscribing regular polygons in a unit circle (which has an area of π) can be used to estimate the value of π, by generating areas that approach π. For example, one group used the sliders to compare the area of the unit circle with a regular polygon when *n* = 3 is shown in Figure 7.

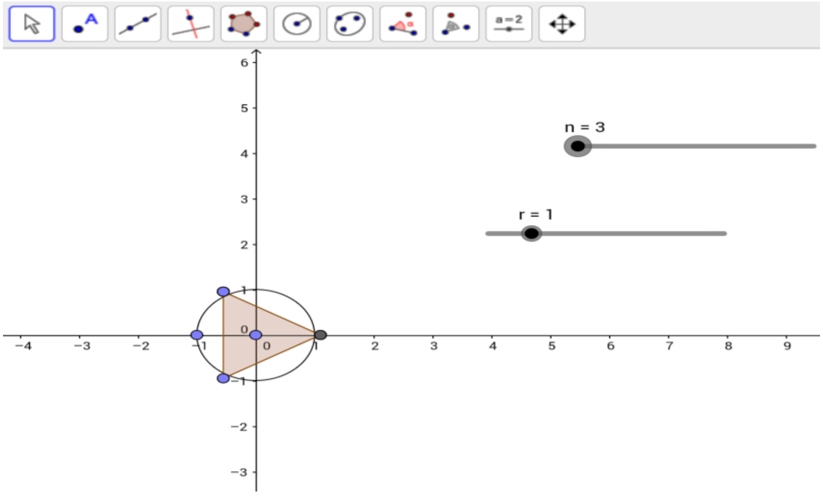


*Figure 7*. Area of a triangle inscribed in a unit circle

GeoGebra supported the group discussions of the PSTs enabling them to understand the area of a circle which led to the estimate of the decimal value of π. The PSTs were guided through the process of finding the area of a regular triangle, so that they could replicate the process. As the PSTs conducted their investigation, the teacher educator emphasised that increasing *n* causes the regular polygons to become more "circle-like," which is why their areas approach the area of the unit circle, which is known to be π. Seven PSTs, however, were unable to identify the approximate value of π as they were not proficient in using GeoGebra to draw the inscribed regular polygons in the unit circle. They had particular difficulty with creating the sliders (a graphical representation of a free number that can be used to vary the number of sides of the inscribed polygon) for n and r as shown in Figure. PST 5 stated:

*To be honest the lesson was completed without understanding how to find the value of π.* (PST 5).

PST5's comment highlighted that difficulty of the software application to create an inscribed polygon with a varied number of sides within a unit circle leads to difficulty in finding the value of π and understanding the concept. This could also be a problem in creating the algebraic relationship between the number of sides (n) and the radius (r).

A summary of the observation results to Teacher Educator 1 and Teacher Educator 2 is shown in Table 3 and Table 4 respectively with modal scores for each TPACK construct along with descriptions of observed practices.

Table 3

*Teacher Educator 1 TPACK as Observed in the Classroom*

| TPACK Constructs | Mode | Description |
|---|---|---|
| TK | 3 | The teacher educator used MSM efficiently |
| CK | 3 | The teacher educator demonstrated the specialist knowledge to teach the content |
| PK | 3 | The group work was efficient in engaging PSTs during the lesson |
| PCK | 3 | The topic was appropriate to be taught in groups involving peer discussion |

| | | |
|---|---|---|
| TPK | 3 | The technology selected was effective in stimulating PSTs' peer discussion |
| TCK | 3 | The technology was appropriate to teach the graph of a logarithmic function |
| TPACK | 3 | The lesson was delivered effectively and in harmony with the selected technology (MSM) and pedagogy involving group work |

***1 = not observed, 2 = partly observed, 3 = observed***

As Table 3 shows, the technology selected was used effectively in teaching about the graphs of logarithmic functions. The technology facilitated PSTs' group discussions. This particular lesson was engaging. The PSTs engagement was evidenced from the teacher educators' TPK as shown in Table 3 on which the technology selected was effective in prompting the PSTs' opportunities to discuss the mathematics concepts with their colleagues.

Table 4

*Teacher Educator 2 TPACK as Observed in the Classroom*

| TPACK constructs | Mode | Description |
|---|---|---|
| TK | 2 | GeoGebra was appropriate and the teacher educator used the technology efficiently with some limitation in understanding PSTs' prior knowledge to GeoGebra |
| CK | 3 | The teacher educator was confident while explaining the content |
| PK | 3 | PSTs were grouped for discussion and the teacher educator was moving around helping each group |
| PCK | 3 | The group work supported learners to discuss concepts and come up with solutions |
| TPK | 3 | The technology supported learners to collaborate and hence supported the group work discussion |
| TCK | 2 | Even though some PSTs found it was challenging to use GeoGebra, the technology was appropriate to teach the area of the circle based on inscribed regular polygon and to estimate the value of π |
| TPACK | 2 | Although some PSTs were challenged in using GeoGebra to estimate the value of π, the technology facilitated the group work discussion of estimating the area of the unit circle |

***1 = not observed, 2 = partly observed, 3 = observed***

As Table 4 shows, the teacher educator was observed appropriately selecting a technology, which fitted with the pedagogy and the content selected to engage the learners. The technology supported group discussion to develop an understanding of the area of a circle and estimate the value of π. The use of GeoGebra helped the teacher educator shift to a more student centred approach. A few PSTs were unable to use the software efficiently and so GeoGebra did not support them to estimate the value of π. However, they learn from each other during the group discussion. Technology use by the teacher educators shifted to a more learners' centred approach which helped the pre-service teachers actively involved in the lessons, even when their knowledge of the technology was not strong.

In addition, the teacher educators and PSTs were asked to share their experiences of technology integrated lessons after the PD program. The question was "What did you think of the lessons you taught?" The reflections of the teacher educators provided in the interview are summarised in the following section categorised as aspects they liked and the aspects they did not like. Table 5 and Table 6 provides a summary of the teacher educators' and PSTs' feedbacks on these lessons respectively.

Table 5

*Reasons for which the Teacher Educators Liked and Disliked Technology Integrated Lesson*

| Liked | Disliked |
|---|---|
| • It engages learners because learners were doing activities by themselves and at their own pace ( Teacher Educator 1)<br>• The approach was different from the usual Though it requires preparation in advance, the process gave more authority to the learners than the teacher educator (Teacher Educator 2)<br>• Gives the opportunity to try our skill in teaching a particular mathematics concept with a different approach (Both Teacher Educator 1 and 2)<br>• It creates the opportunity for PSTs to discuss with colleagues (Both Teacher Educator 1 and 2) | • Preparation of the lesson required ample time as it is not the one I have practiced before (Teacher Educator 1)<br>• They did not like lessons, which involved software unfamiliar to PSTs (Teacher Educator 2)<br>• It was difficult to use some of the applications of the software including creating inscribed circles (Teacher Educator 2) |

It is evident from Table 5, that in spite of the fact that there were some limitations to the teacher educators' use of technology in teaching mathematics they appropriately selected a technology which facilitated the PSTs engagement in the lessons.

Results from interviews and observation sessions indicated that overall PSTs were positive about the technology integrated lessons. The aspects PSTs most liked and disliked are summarised in Table 6.

Table 6

*Reasons for which PSTs Liked or Disliked the Lessons*

| | Reasons | PST |
|---|---|---|
| Liking | I liked the opportunity to be involved in the learning process by my own pace and time | 1,4 |
| | The process simplifies concepts. For example, while learning the graphs of logarithmic function using MSM, the process supported us to graph all logarithmic functions on the same x-y axis with different colours | 3 |
| | I liked lessons, which included motions. For example, while learning logarithmic functions, the software supported us to show the shapes of graphs of logarithmic functions as the base b varies without the need to sketch multiple graphs<br>The combined lesson of an overhead projector and learning with computers was more engaging and thought provoking. | 2 |
| Disliking | The software that we were not familiar with [no time] to play around. Lessons involving software, which we were not familiar, were boring. It took time to play around with the software rather than learning the concept | 5 |

As shown in Table 6, the PSTs enjoyed the technology integrated lessons which involved them in classroom activities and gave them an opportunity to play around with technologies. They were, however, unhappy with lessons that were dominated by the teacher educator and lessons involving technologies that demanded more advanced technical skills than they had.

## 7. Discussion and Conclusion

This study described and illustrated the use of two free software applications to teach specific mathematics topics for understanding. The software provided a variety of utilities that were able to engage PSTs to relearn, and reorganise their knowledge mathematics. Although the interviewed PSTs recognised benefits of the software to learn mathematics, one of them believed that the topics should be taught with traditional methods before being explored using the technology. [7] claimed that such a preference could be due to the difficulty teachers have in adopting appropriate pedagogies for particular software. Given the inexperience of teaching with a technology of the teacher educator in this study, inexpert pedagogy may underpin this PST's opinion as well as her difficulty in understanding what was going on exactly.

During the lesson observation, PSTs readily used MSM to visualise graphs and identify their properties. The 'animate' facility in MSM allowed them to display the graphs as desired based on changing parameters, and helped to facilitate discovery-style lessons [31]. Similar findings have been reported by [4] which showed that facilitating students' visualisation through the application of technology has an increasing importance because students can explore, solve, and communicate mathematical concepts. In addition, the activities supported by visualisation improved PSTs learning of the mathematics concept [34].The MSM software capabilities were particularly important for the chosen mathematics content because difficulties had been identified in relation to students' ability to distinguish the graphs of different logarithmic functions [8]. In addition, MSM supported the PSTs to describe the graphs of logarithmic functions and apparently supported their understanding of the topic. The usefulness of MSM in learning about the graphs of logarithmic function appeared due to its ability to facilitate the learning processes by making it easier to accurately produce graphs of logarithmic functions on the same axes and support PSTs to notice the effect of altering particular parameters (in this case the base of a logarithmic function) on the properties of the function's graph. Consistent with previous studies such as [34], MSM supported PSTs to understand the mathematical concepts better and actively engaged PSTs through interaction with a computer and their peers that are not obtained with conventional teaching (see Table 4).

Similarly, this study highlighted opportunities that GeoGebra can offer for the teaching of a geometry concept. The teacher educator used GeoGebra to facilitate PSTs' learning to estimate the decimal value of π and the area of a unit circle using a regular polygon inscribed in the circle. The teacher educator used the technology to facilitate a discovery method. It encouraged the PSTs to take a more active independent role in their learning by answering a series of questions designed by the teacher educator to guide them to the intended outcome, which is similar to dynamic geometry software use suggested by other studies. For example, [20] and [26] showed GeoGebra fosters experimental, problem-oriented, independent learning, and discovery learning of mathematics. In this study, the PSTs independently discovered that as the number of sides of the inscribed regular polygon increased (such as n = 200), the approximate value of area of a unit circle and the value of π approaches to 3.14 (as an area of a circle is $\pi r^2 = \pi$ when $r = 1$). In spite of the fact that GeoGebra supported the 24 PSTs to learn the concept easily, seven PSTs were observed to find it challenging to use the software to sketch an inscribed regular polygon with $n$ sides in a unit circle. Particularly, creating the 'slider' (a visual representation of a number and avoids frequent tying of the value of a variable in an equation) which required entering algebraic commands in the input box. As a result, the PSTs were unable to create an inscribed polygon with a varied number of sides within a unit circle, which prevented them from learning to estimate the value of π. This finding is similar to [12] who stated that students without previous experience would hardly enter algebraic commands in the input box of GeoGebra and as a result they may feel quite at a loss of what to do. In addition, the importance of knowledge of specific software for effective ICT integrated teaching is emphasised by [24]. This implies that teacher educators should understand their PSTs' prior knowledge of the technology, as well as the use of specific software to teach a mathematical concept because effective teaching with technology requires understanding learners' prior knowledge about the content and the technology used [24]. Moreover, the PSTs' skill with a particular ICT was a facilitating factor for ICT integrated mathematics teaching as suggested by [30]. Studies have indicated that GeoGebra enabled students to discover mathematical concepts independently (see [19 and [26]). In this study, however, the results indicated that the full potential of GeoGebra to facilitate a discovery approach could be effectively realised when learners had prior knowledge of how to use GeoGebra. As [7] showed, success in integrating technology in mathematics lessons is not only dependent upon knowledge of the software that is used by mathematics teachers for their personal use, but technology technical skills related are equally important for proper implementation of technology in teaching. Studies have shown that having the knowledge of the applications of various technologies is a facilitative factor to use ICT in teaching effectively (see [13 and [30]). Particularly, [13] showed that ICT should not just be regarded, as a replacement for existing teaching methods; rather, teachers, as well as their students, must have the required knowledge to use a particular technology in teaching and learning. The current study supports these ideas. A study conducted in Sub Saharan African countries also indicated that having the technical knowledge of technologies is an important contribution to the effective use of technology in teaching [16]. Concerning the findings of this study, PSTs' limited prior knowledge of GeoGebra influenced their effective use of the technology. However**,** the results further showed that the GeoGebra software was appropriate, relevant and highly effective for most PSTs for developing and understanding of the selected topic.

Although GeoGebra and MSM facilitated discovery learning, the teacher educators' knowledge of technology, pedagogy and content played an important role in PSTs' learning of the concepts (see Table 3). In this regard, studies have shown that having integrated knowledge of ICT, pedagogy and content of teachers' leads to an effective teaching approach using technology see [24] and [29]). Similarly to many other studies such as [6], [12], [34], and [39], in this study, GeoGebra and MSM make the lessons more engaging by enabling the tasks based on trial, improvement and experimentation. The software also fostered PSTs' idea exchange by providing support for exploration and consequent sharing of discoveries. Equally, the effective use of GeoGebra and MSM by the teacher educators was helpful for the PST's future teaching. Other studies also indicated that teacher educators, who use ICT for the enhancement of the learning process of PSTs, also showed their learners at the same time how ICT can be used in their future teaching (see [35] and [32]).

To sum up, this study has provided an example of how GeoGebra and MSM can be used to explore basic mathematics concepts. These software applications were pivotal mechanisms through which the teacher educators shifted their focus from direct teaching to PSTs learning through being actively engaged in the lessons. The findings from this study are significant to those looking to improve mathematics teaching with GeoGebra and MSM recognising that this can influence their learners' understanding of mathematical concepts through visual means as students cannot always focus on the lessons for a long time by the use of traditional teaching methods. This result is similar to previous studies which have revealed that activities supported by visualisation can improve learning mathematics (see [4] and [34]). In addition, using GeoGebra and MSM can create opportunities for the teacher educator and PSTs to share their experiences at the international level via user communities, for example, in using GeoGebra [20], thus, the impact of a user community is worthy of further investigation. These software capabilities were particularly important for the chosen mathematics content because difficulties had been identified in relation to the PSTs ability to distinguish these concepts. This study further highlighted the use of specific software to teach a specific mathematical concept and software with similar capabilities could be useful for other mathematical concepts. Particularly, it has demonstrated the potential of freely available software to help teachers in developing countries and other contexts where resources are limited and the need for prior knowledge to use particular software/technology for teaching and learning.

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## Appendix

### CLASSROOM OBSERVATION CHECKLIST

Teacher educator code: ________________________
Lesson Topic: ______________________
Name of College of Teacher Education:_____________________
Technology used: _____________________________
Classroom Context Description:

______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
________

**Note:** 3=observed; 2 = partly observed and 1=Not observed

| | | 3 | 2 | 1 | Example of fully/observed or partly observed practice |
|---|---|---|---|---|---|
| **Content Knowledge (CK)** | | | | | |
| 1 | Clearly introduced the topic and learning goals | | | | |
| 2 | Sufficient knowledge of the lesson | | | | |
| 3 | Demonstrates confident in mathematical concepts related to lesson | | | | |
| 4 | Uses appropriate materials in relation to the given lesson being taught | | | | |
| | | | | | |
| **Pedagogical Knowledge (PK)** | | | | | |
| 5 | Engage pre-service teachers in exploring real-world issues and solving authentic problems using teaching resources. | | | | |
| 6 | Address the diverse needs of all learners by using learner-centered strategies | | | | |
| 7 | Providing equitable access to appropriate resources | | | | |
| | | | | | |
| **Technological Knowledge (TK)** | | | | | |
| 8 | Teacher demonstrates developed knowledge in selecting technology skills | | | | |
| 9 | Demonstrate fluency in the transfer of the used technology knowledge to new situations | | | | |
| 10 | Demonstrate knowledge on effective combination of learning support tools | | | | |
| | | | | | |
| **Pedagogical Content Knowledge (PCK)** | | | | | |
| 11 | Possess the ability to integrate teaching approaches that arouse pre-service teachers' creativity | | | | |
| 12 | Apply teaching approaches which give more authority to pre-service teachers in solving mathematics problem | | | | |
| | | | | | |
| **Technological Pedagogical Knowledge (TPK)** | | | | | |
| 13 | Engage pre-service teachers in the pedagogy used in learning activities | | | | |
| 14 | Use the technology used to help pre-service teachers to collaborate | | | | |
| | | | | | |
| **Technological Content Knowledge (TCK)** | | | | | |
| 15 | Clear link between technology and the mathematics knowledge | | | | |
| 16 | Design relevant learning experiences that incorporate the technology used to promote pre-service teachers learning | | | | |
| 17 | Introduction of fundamental concepts by technology incorporation | | | | |
| | | | | | |
| | | | | | |

| **Technological, Pedagogical and Content Knowledge (TPACK)** | | | | | |
|---|---|---|---|---|---|
| 18 | Proper choice of technology in relation to mathematics concept and pedagogy | | | | |
| 19 | Clearly integrate the components of TPACK to promote creative thinking in pre-service teachers | | | | |
| 20 | Apply TPACK to promote pre-service teachers' reflection and to clarify pre-service teachers' conceptual thinking. | | | | |

**Final Note:**